



## Catalysis and autocatalysis of chemical synthesis and of hadronization

C.G. Vayenas<sup>a,b,\*</sup>, A.S. Fokas<sup>c,d,\*\*</sup>, D. Grigoriou<sup>a</sup><sup>a</sup> LCEP, Department of Chemical Engineering, 1 Caratheodory St., University of Patras, Patras GR-26504, Greece<sup>b</sup> Division of Natural Sciences, Academy of Athens, 28 Panepistimiou Ave., GR-10679 Athens, Greece<sup>c</sup> Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK<sup>d</sup> Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089-2560, USA

## ARTICLE INFO

## Article history:

Received 24 June 2016

Received in revised form

27 September 2016

Accepted 28 September 2016

Available online 18 October 2016

## Keywords:

Ammonia synthesis

Homogeneous H<sub>2</sub> oxidation

Proton synthesis

Hadronization catalysis

Charged promoters

Autocatalysis

## ABSTRACT

We compare the thermodynamics and catalysis of two important exothermic chemical reactions, the homogeneously catalyzed chain reaction of H<sub>2</sub> oxidation and the heterogeneously catalyzed ammonia synthesis, with those of an important exothermic physical reaction, namely the proton or neutron synthesis from elementary particles, i.e. quarks or relativistic neutrinos, a process known as hadronization. We show that, surprisingly, hadronization has several similarities both with homogeneous autocatalytic chain reactions, as well as with heterogeneously catalyzed ammonia synthesis. In NH<sub>3</sub> synthesis, the presence of electrical charge, namely alkali promoters or protons or electrons on transition metals, enhances the formation of catalytically active intermediate NH species, while in hadronization, free electrons or positrons lower the activation energy and facilitate the formation of catalytically active bosonic  $e^\pm$ -neutrino intermediates. These entities liberate highly active neutrinos, creating, via their relativistic mass increase, a strong local gravitational field with all the mass-producing properties of the Higgs field. The main analogies and differences between the electric field effects in chemical catalysis and the electric and gravitational field effects in hadronization catalysis are identified and briefly discussed.

© 2016 Elsevier B.V. All rights reserved.

## 1. Introduction

Exothermic reactions, favored thermodynamically at lower temperatures [1] and thus requiring the presence of a catalyst, play an important role both in chemistry and in high energy physics. For example the homogeneous gas-phase oxidation of H<sub>2</sub> to H<sub>2</sub>O, via a free radical chain reaction, is strongly catalyzed by OH radicals. The elucidation of this homogeneous catalytic system played a key role in understanding the three explosion limits of H<sub>2</sub>-O<sub>2</sub> mixtures [2,3] which was quite important for several industrial applications. In the area of heterogeneous catalysis, as another example, the Haber–Bosch ammonia synthesis process has had an enormous impact on our survival during the last 100 years since it is of paramount importance in the chemical industry mainly for fertilizers production [4,5]. This reaction is equilibrium limited and is favored by low temperatures and high operating pressure

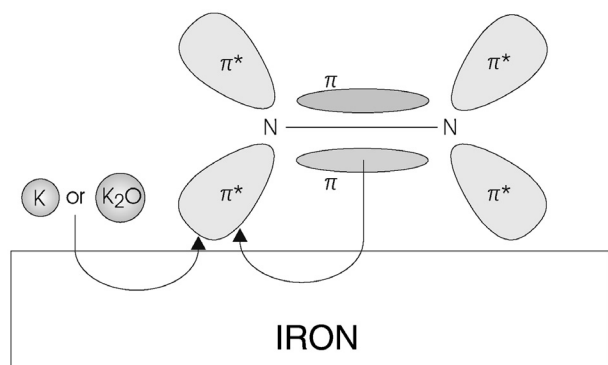
[1,4–6]. Since the pioneering work of Haber and Bosch the industrial ammonia synthesis has been carried out over potassium-promoted Fe catalysts at pressures up to 300 bar [4]. More recently, alkali-promoted Ru catalysts have been shown to be more active and this allows for a reduction in operating temperature and pressure [4,5].

As shown in Fig. 1, the role of alkali promoters is to enhance the dissociation of molecularly adsorbed N<sub>2</sub> via the donation of electrons to the antibonding  $\pi$ -electrons of N<sub>2</sub>. In addition to alkalis, protons supplied electrochemically via a H<sup>+</sup> conducting solid electrolyte [6], have also been shown to act as promoters for the electrochemical promotion [7–10] of ammonia synthesis on a fully promoted Fe-based industrial catalyst [6] by enhancing the dissociative adsorption of nitrogen. In a very recent study, Hosono and coworkers [5] have used a 12Ca<sub>0.7</sub>Al<sub>2</sub>O<sub>3</sub> electride (C12A7:  $e^-$ ), the first room temperature stable electride, which functions as an efficient catalyst support and electronic promoter for Ru catalysts [5]. This electride has a unique crystal structure consisting of a positively charged framework having the chemical formula [Ca<sub>24</sub>Al<sub>28</sub>O<sub>64</sub>]<sup>4+</sup> and four extra-framework electrons, accommodated in the cages as the counter-ions [5]. It is worth noting that ex situ added alkali promoters, electrochemically supplied protons or (C12A:  $e^-$ ) electron donating supports, all produce a Fe or Ru catalyst surface with an extremely low ( $\sim 2.4$  eV) work function, comparable to that of potassium metal, which are highly suitable

\* Corresponding author at: LCEP, Department of Chemical Engineering, 1 Caratheodory St., University of Patras, Patras GR 26504, Greece.

\*\* Corresponding author at: Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 0WA, UK.

E-mail addresses: [cgvayenas@upatras.gr](mailto:cgvayenas@upatras.gr) (C.G. Vayenas), [T.Fokas@damtp.cam.ac.uk](mailto:T.Fokas@damtp.cam.ac.uk) (A.S. Fokas).



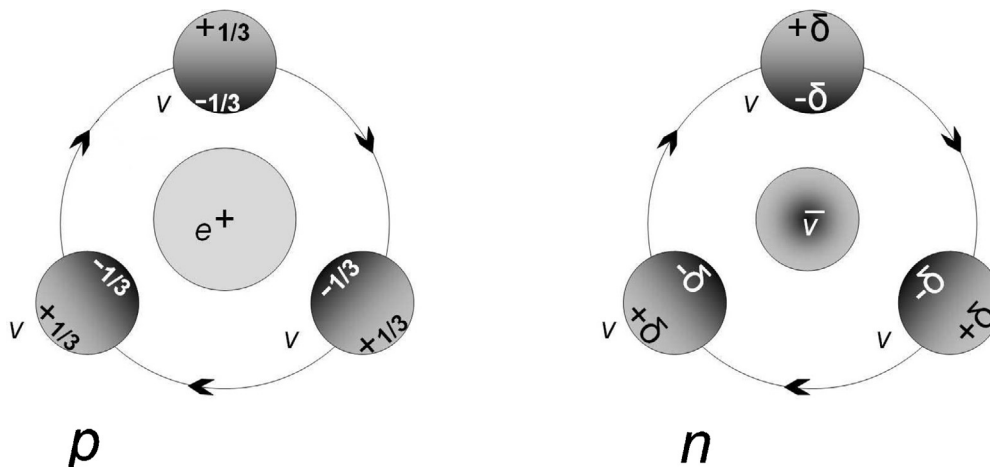
**Fig. 1.** Spatial distribution of the main orbitals of  $N_2$  involved in molecular chemisorption on iron promoted by potassium (K or  $K_2O$ ). Arrows indicate the direction of the transfer of electron density [8]. Reprinted with permission from Springer.

for electron donation. The promoting action of electrons from the electrode support is described schematically in Fig. 2.

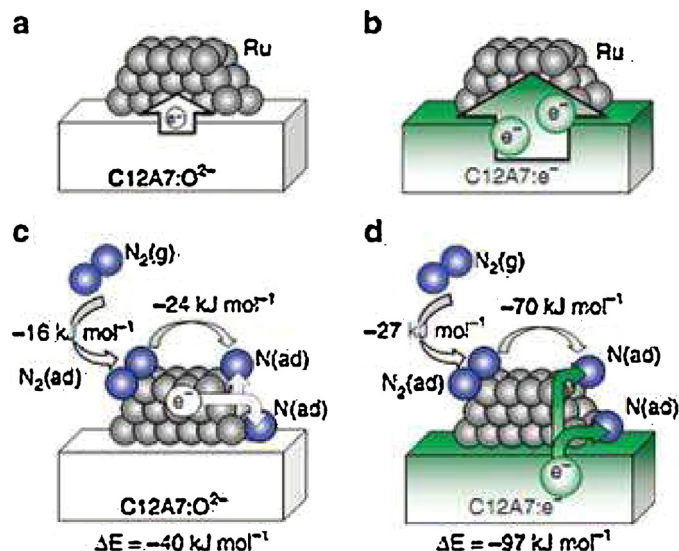
In the present work, surprisingly, electrons (and positrons) are also shown to act as very efficient catalysts for hadronization, which is the formation of hadrons, i.e. baryons and mesons. Of particular interest is baryogenesis, i.e. the synthesis of baryons, such as protons and neutrons, which are composite particles containing three much lighter particles known as quarks [11–14]. This is an extremely important reaction, also known as quark-gluon plasma condensation, which apparently took place some  $10^{-5}$  s after the big bang [14] and led to the creation of visible mass, i.e. protons, neutrons and other baryons [11–14].

In what follows we will discuss for the first time the catalytic action of electrons and positrons on hadronization, i.e. on the association (more commonly known as confinement or condensation [14]) of fast light particles, such as quarks, gluons [14], or perhaps simply neutrinos [13,15], to form heavier particles such as protons and neutrons.

It will be shown that the catalytic action of  $e^\pm$  in hadronization bears similarities both with the catalytic role of OH radicals in homogeneous gas phase oxidation catalysis, i.e., in free radical chain reactions [2,3], as well as with the catalytic role of  $e^-$  or  $H^+$  promoted metal catalysts in ammonia synthesis. Since in recent years mass generation is also commonly associated with the action of a field known as the Higgs field [11,12], we will also briefly discuss the apparent connection between this field and the catalytic action of electrons and positrons on hadronization.



**Fig. 3.** Schematics of a proton and a neutron according to the rotating neutrino model, showing the central positron in the case of the proton and the electric dipoles induced on the rotating neutrinos [17].



**Fig. 2.** Ab initio simulations of  $N_2$  interaction with the Ru/C12A7 catalysts. Character of the charge redistribution between C12A7 substrate and deposited Ru clusters for the stoichiometric (a) and electride (b) C12A7. (c, d) Adsorption energies of  $N_2$  on C12A7-supported Ru, charge transfer in the process of  $N_2$  dissociation ( $N_2(g) + Ru \rightarrow 2N(ad) + Ru$ ) and the corresponding energy gain (E). In Ru/C12A7:O<sub>2</sub> system (c),  $N_2$  and N accept electron charge from the Ru cluster, making it positively charged. In Ru/C12A7:e (d), the electron charge is transferred from the substrate, leaving the Ru cluster nearly neutral.  $N_2(g)$ ,  $N_2(ad)$  and  $N(ad)$  represent  $N_2$  in gas phase, adsorbed  $N_2$ , and adsorbed nitrogen atom, respectively [5]. Reprinted with permission from Nature materials.

### 1.1. Hadron structure modeling

In a book [13] and several recent papers [15–18] we have presented a simple Bohr-type three rotating neutrino model for baryons (Fig. 3) and have shown that the rest mass,  $m_0$ , of the three constituents of protons and neutrons, termed quarks, is the same with the rest mass of neutrinos, which is much smaller than the rest mass of protons and neutrons (Table 1).

The analysis of this model involves the following steps. First, it is shown [13,15] that the mass,  $m$ , of the composite rotational state is obtained via the equation

$$mc^2 = 3\gamma m_0 c^2; \quad m = 3\gamma m_0, \quad (1)$$

derived from energy conservation, where  $m$  is the mass of the composite state (e.g. a proton),  $\gamma = (1 - v^2/c^2)^{-1/2}$  is the Lorentz

**Table 1**

Rest masses ( $m_0$ ) of neutrinos, electrons, protons and neutrons. The latter two are composite structures, so their mass  $m$  can be obtained from  $m_0$  provided that the Lorentz factor  $\gamma$  can be computed.

Particle	Rest mass (eV/c <sup>2</sup> )	Rest mass (kg)	( $m/3m_0$ )( $=\gamma$ )
$\nu$ (neutrino)	0.04372( $\pm$ 0.01)	$7.794 \times 10^{-38}$	
$e^\pm$ (electron or positron)	$0.511 \times 10^6$	$9.109 \times 10^{-31}$	
p (proton)	$938.272 \times 10^6$	$1.6726 \times 10^{-27}$	$7.153 \times 10^9$
n (neutron)	$939.565 \times 10^6$	$1.6749 \times 10^{-27}$	$7.163 \times 10^9$

factor,  $v$  is the speed, and  $m_0$  is the rest mass of the constituent particles e.g. neutrinos. Second,  $\gamma$  can be computed by solving the two equations of the Bohr-type model which utilizes gravity rather than electrostatic attraction as the centripetal force. These two equations are the relativistic equation for circular [13,15,19,20] motion

$$\frac{\gamma m_0 v^2}{r} = \frac{G m_0^2 \gamma^6}{\sqrt{3} r^2}; \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{1/2}, \quad (2)$$

and the de Broglie wavelength,  $\lambda$ , equation

$$\lambda = r = \frac{\hbar}{\gamma m_0 v}. \quad (3)$$

The right-hand side of the first equation (2) is the universal gravitational Law of Newton, but with the important modification that it involves the gravitational rather than the rest masses of the two particles. According to the equivalence principle, gravitational mass equals inertial mass and the latter equals  $\gamma^3 m_0$  as already shown in 1905 by Einstein for linear motion [21] and confirmed more recently for arbitrary motion as well [13,15]. Eqs. (2) and (3) can be solved analytically for large  $v$  and the solution for  $\gamma$  is

$$\gamma = 3^{1/12} \left(\frac{m_{Pl}}{m_0}\right)^{1/3}, \quad (4)$$

where  $m_{Pl} = (\hbar c/G)^{1/2}$  is the Planck mass.

Upon combining (1) and (4) one obtains from first principles an expression for the baryon mass, namely

$$m = 3^{13/12} (m_{Pl} m_0^2)^{1/3}. \quad (5)$$

By setting  $m = 939.565 \text{ MeV}/c^2$  (the neutron mass, Table 1) one computes  $m_0 = 0.04372 \text{ eV}/c^2$ , which surprisingly lies within the mass range of the heaviest neutrino ( $0.051 \pm 0.01$  [22,23]). This confirms that the rest mass of quarks is that of neutrinos and that quarks may be viewed as electrically polarized neutrinos [13,15,17,18].

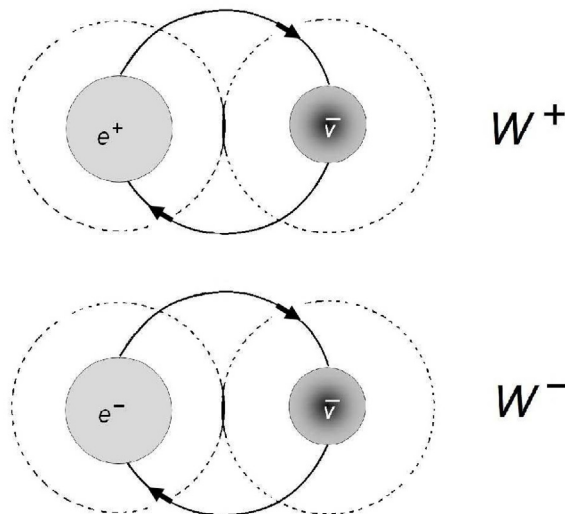
It may appear surprising to a catalysis reader that three neutrinos of initial rest mass  $\sim 0.13 \text{ eV}/c^2$  can form a neutron with a mass which is a factor of  $7.163 \times 10^9$  larger, i.e.  $939.565 \text{ MeV}/c^2$ . However, it should be recalled that energy rather than mass is generally conserved, and in the present case it is the huge rotational kinetic energy that the neutrinos acquire in their rotational motion which is the major part of the rest energy of the composite neutron structure formed. The autocatalytic aspects of this mass generation scheme which produces protons and neutrons from almost massless neutrinos is discussed in Section 2.1.2.

As shown in Table 1, it turns out that the relativistic velocities which these rotating particles acquire in the interior of baryons, such as protons and neutrons, are so high, that the  $\gamma$  values computed from first principles (Section 2 and Appendix A) are of the order of  $10^{10}$ , and remarkably correspond to the ratios of the masses of baryons and neutrinos (Table 1). The model, which contains no adjustable parameters, is in good agreement with both the thermodynamics and the kinetics of baryogenesis. The striking similarity of the thermodynamics and kinetics of this condensation reaction with the thermodynamics and kinetics of exothermic chemical condensation or synthesis reactions, such as ammonia synthesis, is

**Table 2**

Basic thermodynamic properties of the synthesis of  $\text{NH}_3$  and a neutron;  $T_{cr}$  is the temperature where  $\Delta G = 0$ .

Reaction	$\Delta H$ (kJ/mol)	$\Delta S$ (J/mol K)	$T_{cr}$ (K)	Ref.
$1/2 \text{N}_2 + 3/2 \text{H}_2 \rightarrow \text{NH}_3$	-45.86	-99.1	$4.6 \times 10^2$	[1]
$3\nu \rightarrow n$	$-6.02 \times 10^{10}$ ( $= -625 \times 10^6 \text{ eV/atom}$ $= -(2/3)m_n c^2$ )	-9.20	$6.56 \times 10^{12}$	[13,16]



**Fig. 4.** Top: Gravitationally confined rotating positron-anti-neutrino and electron-anti-neutrino models corresponding to the  $W^+$  and  $W^-$  bosons [17]. The mass for each constituent decay product, computed with  $m_\nu = 0.0437 \text{ eV}/c^2$  [13,15,17,18], is  $81.74 \text{ GeV}/c^2$ . Exact agreement with the experimental mass of  $80.42 \text{ GeV}/c^2$  is obtained for  $m_\nu = 0.04164 \text{ eV}/c^2$ . Dotted lines show the Compton wavelength limits.

shown in Table 2 which compares the  $\Delta H$ ,  $\Delta S$  and  $T_{cr}$  ( $= -\Delta H/\Delta S$ ) values of the ammonia synthesis from  $\text{N}_2$  and  $\text{H}_2$ , with the corresponding values for the synthesis of a neutron from three neutrinos [13,15,16].

It has been recently shown that neutrinos can also combine with electrons or positrons to form rotating relativistic states [17,18]. The mass of each component of the neutrino-electron rotating structure, which is analytically computed from first principles, shown in Fig. 4 is, remarkably,  $81.74 \text{ GeV}/c^2$ , i.e. a factor of 80 heavier than protons and neutrons and practically equal to the mass of the  $W^\pm$  boson [17]. This boson, as well as the  $Z^0$  and H (Higgs) bosons, can liberate upon decomposition, highly energetic neutrinos with energies well above the activation energy for hadronization. One may thus consider such boson structures as catalysts for hadronization and the positrons and electrons constituents as promoters (Table 3).

**Table 3**

Analogies between free radical chain reactions, ammonia synthesis and hadronization.

Reaction	Catalysts	Promoter	Action
Free radical chain reaction (e.g. $\text{H}_2$ oxidation)	OH		Lowering of activation energy
$\text{NH}_3$ synthesis	Fe, Ru	$e^-$ , p, alkalis	Lowering of activation energy
Hadronization	$e^-$ , $e^+$ , $W^\pm$ , $Z^0$ , H bosons (composite $e^\pm - \nu_e$ structures)		Lowering of activation energy

**Table 4**Catalysis of ammonia synthesis: activation energy  $E$  (kJ/mol) of  $\text{NH}_3$  synthesis and  $\text{N}_2$  desorption. Effect of  $e^-$  and  $\text{Cs}^+$  on supported Ru catalyst [5];  $T$  range 613–673 K

Catalyst	$\text{NH}_3$ synthesis	$\rho_{\text{Cs}}$	$\text{N}_2$ exchange ( $\text{N}_2$ desorption)	$\rho_e$	$\text{N}_2$ TPD ( $\text{N}_2$ desorption)
Ru/MgO	118		154		
Ru-Cs/MgO	99	3.6	139	2.85	137
Ru/C12A7: $e^-$	49	13.1	58	18.3	64

## 1.2. Catalysis of the $\text{NH}_3$ synthesis by electrons, protons and alkalis

Table 4 summarizes the recent results of [5] regarding the effect of Cs promoter and electride support on the activation energies for  $\text{NH}_3$  synthesis,  $\text{N}_2$  exchange and  $\text{N}_2$  desorption. One observes the pronounced decrease in activation energy caused by the addition of Cs and by the addition of  $e^-$  via the electride support.

Denoting by  $\mathcal{R}$  the catalytic rate of  $\text{NH}_3$  synthesis we have computed via the law of Arrhenius the rate enhancement ratio,  $\rho$ , [7,8]; The computed values shown in the table at  $T = 633$  K, are computed from the following equations:

$$\begin{aligned}\rho_{\text{Cs}} &= \frac{\mathcal{R}_{\text{Ru-Cs/MgO}}}{\mathcal{R}_{\text{Ru/MgO}}} = \frac{\exp[-E_{\text{Ru-Cs/MgO}}/RT]}{\exp[-E_{\text{Ru/MgO}}/RT]} \\ &= \exp[(E_{\text{Ru/MgO}} - E_{\text{Ru-Cs/MgO}})/R633 \text{ K}] \\ &= 19 \text{ kJ}/5.26 \text{ kJ} = 3.61,\end{aligned}\quad (6)$$

$$\begin{aligned}\rho_{e^-} &= \frac{r_{\text{Ru}/e^-}}{r_{\text{Ru/MgO}}} = \exp[(E_{\text{Ru/MgO}} - E_{\text{Ru}/e^-})/R633 \text{ K}] \\ &= (69 \text{ kJ}/5.26 \text{ kJ}) = 13.1\end{aligned}\quad (7)$$

Thus, the addition of  $\text{Cs}^+$  and  $e^-$  causes roughly a threefold and thirteenfold increase, respectively, in the rate of ammonia synthesis. Interestingly, this is also the range of  $\rho_{\text{H}^+}$  values for the ammonia synthesis measured in electrochemical promotion studies [6] via the in situ addition of protons to a commercial Fe-based ammonia synthesis catalyst supported on a  $\text{Ca}_{0.1}\text{Zr}_{0.9}\text{O}_{3-\alpha}$  proton conductor [6,8]. The promoting effect of protons and alkalis on the  $\text{NH}_3$  synthesis is commonly attributed to the enhanced chemisorptive binding energy and sticking coefficient of nitrogen on the catalyst surface, while the promoting role of electrons is commonly attributed to the destabilization of the  $\text{N}\equiv\text{N}$  bond due to electron donation to the antibonding  $\pi$ -electrons of  $\text{N}_2$  [4–6,8].

The definition and computation of the above rate enhancement ratio,  $\rho$ , values is useful since it permits a direct comparison with similarly defined  $\rho$  values in hadronization.

## 2. Catalysis by electrons or positrons of mass generation via confinement of fast neutrinos

### 2.1. Hadronization via gravitational confinement of neutrinos

#### 2.1.1. Steady state features

According to the Standard Model of Physics [11,12] hadrons (i.e. baryons, such as protons and neutrons, and also mesons) consist of quarks and antiquarks confined by the Strong Force via the action of gluons. As already noted, according to the recently developed Rotating Neutrino Model (RNM) hadrons comprise a rotating ring of fast relativistic neutrinos confined by the relativistic gravitational force (Fig. 3). The latter is computed via the Newton gravitational law but where one uses gravitational masses,  $\gamma^3 m_0$ , instead of rest masses  $m_0$ . Thus, denoting by  $\ell$  the distance between two particles,

each with rest mass  $m_0$  and with relative speed  $v$ , this force is expressed as

$$F = \frac{Gm_0^2\gamma^6}{\ell^2}, \quad (8)$$

where  $\gamma(=1-v^2/c^2)^{-1/2}$  is the Lorentz factor which approaches infinity for  $v \rightarrow c$ . This expression was already used for a three-particle system in Eq. (2) of the Introduction.

In the RNM a meson is modeled as a pair of two relativistic neutrinos confined in a circular orbit of radius  $r$  by their gravitational attraction, similarly to the  $e^\pm$ -neutrino pair of Fig. 4. This implies

$$\gamma m_0 \frac{v^2}{r} = \frac{Gm_0^2\gamma^6}{4r^2}. \quad (9)$$

As in the Bohr model of the  $H$  atom, a second equation is obtained via the de Broglie wavelength expression. For the ground state this equation yields

$$\lambda = r = \frac{\hbar}{\gamma m_0 v}. \quad (10)$$

Solving Eqs. (9) and (10), one obtains Eqs. (11) and (13), i.e.

$$v \approx c; \quad \gamma = \gamma_{f,2} = 2^{1/3} \left( \frac{m_{\text{Pl}}}{m_0} \right)^{1/3} = 8.234 \times 10^9, \quad (11)$$

where the subscript “ $f$ ” denotes *final*, and the subscript “2” denotes the number of neutrinos involved. Also, as already noted,

$$m_{\text{Pl}} = \left( \frac{\hbar c}{G} \right)^{1/2}, \quad (12)$$

is the Planck mass ( $1.223 \times 10^{28} \text{ eV}/c^2 = 2.1765 \times 10^8 \text{ kg}$ ). Eq. (10) with  $v \approx c$  yields

$$r = r_{f,2} = \frac{\hbar}{\gamma_{f,2} m_0 c} = 0.548 \times 10^{-15} \text{ m}. \quad (13)$$

Due to energy conservation, the mass,  $m_{f,2}$ , of the final state is given by

$$m_{f,2} = 2\gamma_{f,2} m_0. \quad (14)$$

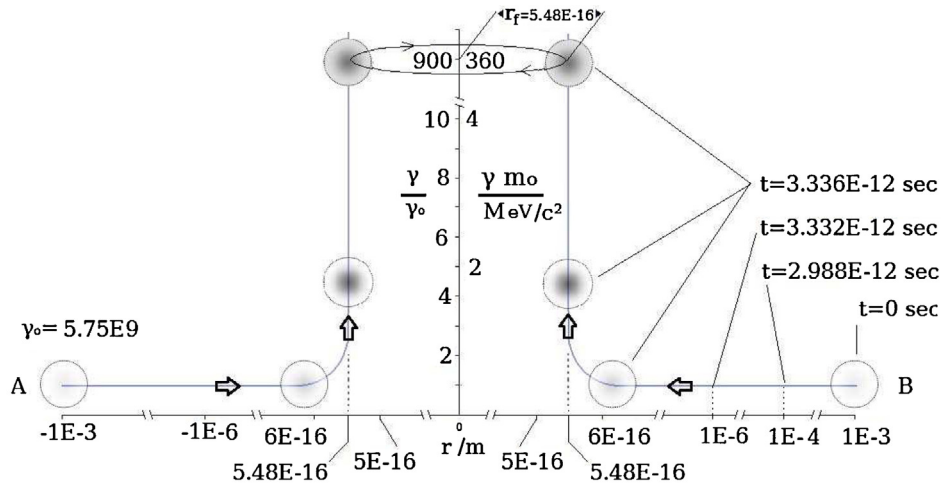
Hence Eq. (11) implies

$$m_{f,2} = 2^{4/3} (m_{\text{Pl}}/m_0^3)^{1/3} = 720 \text{ MeV}/c^2, \quad (15)$$

where we are using, as always,  $m_0 = 0.04372 \text{ eV}/c^2$  for the heaviest neutrino mass [13,15,22,23]. The above computed  $m_{f,2}$  value is in the mass range of  $\rho$  mesons [11–13].

According to the Standard Model, baryons, such as protons and neutrons, consist of three quarks confined via gluons by the Strong Force. The quarks are assumed to have fractional electric charges, i.e.  $2e/3$  for the up quarks and  $-e/3$  for the down quarks [11,12]. According to the RNM model baryons comprise a rotating ring consisting of three neutrinos which are electrically polarized due to the presence of a positron at the center of the ring for proton and a polarized neutrino at the center of neutrons, Fig. 3. The electrostatic forces between the central positron or electrically polarized neutrino and the three rotating neutrinos of the ring are very similar to the charge-induced dipole or induced dipole-induced dipole type forces which are very well known and important in Chemistry but which are rarely used in High Energy Physics. They have a minor effect ( $2\text{--}3 \text{ MeV}/c^2$ ) in the baryon and boson masses computed here





**Fig. 5.** Mass generation mechanism via the creation of a gravitational Higgs-type field by two particles (neutrinos) A and B of rest mass  $m_0$  and initial Lorentz factor  $\gamma_0$ . The figure shows the time and corresponding space (distance) evolution of the particle Lorentz factor  $\gamma (= (1 - v^2/c^2)^{-1/2})$  and thus of the relativistic particle mass  $\gamma m_0 c^2$ .

[13,15,17,18] which are dominated by the relativistic mass,  $\gamma_{f,2} m_0$  or  $\gamma_{f,3} m_0$ , of the rotating neutrinos.

In the case of a three neutrino ring, Fig. 3, the starting Eq. (2) becomes Eq. (1), i.e.

$$\gamma m_0 \frac{v^2}{r} = \frac{G m_0^2 \gamma^6}{\sqrt{3} r^2},$$

where the factor  $\sqrt{3}$  results from the symmetric particle geometry. Solving this equation in conjunction with the de Broglie wavelength equation (3) (or (10)) one obtains

$$\gamma = \gamma_{f,3} = 3^{1/12} \left( \frac{m_{Pl}}{m_0} \right)^{1/3} = 7.163 \times 10^9. \quad (16)$$

Hence

$$m_{f,3} = 3 \gamma_{f,3} m_0 = 3^{13/12} \left( \frac{m_{Pl}}{m_0} \right)^{1/3} = 939.5 \text{ MeV}/c^2, \quad (17)$$

where we have used again  $m_0 = 0.04372 \text{ eV}/c^2$ .

As already noted, with this choice of  $m_0$  the computed  $m_{f,3}$  value coincides exactly with the mass of a neutron ( $939.565 \text{ MeV}/c^2$ ) [11–13].

### 2.1.2. Dynamic features of hadronization and autocatalysis

In order to study the dynamics of hadronization, namely meson production from two neutrinos, similar to the  $\nu_e - e$  structure of Fig. 4, or baryon formation from three neutrinos, Fig. 3, we examine first the mutual linear approach of two neutrinos attracted by their gravitational attraction (Fig. 5). We seek to determine the probability of a successful approach leading to the formation of a two-particle circular rotating meson-type structure.

For this formation to occur there are two requirements: First, that the two particles approach each other within a distance  $r_{f,2}$  (Eq. (13)) which is the minimum approach distance allowed by the Compton wavelength requirement or equivalently by the Heisenberg uncertainty principle [11–13]. And second, that the associated speed  $v_f$  and thus associated Lorentz factor  $\gamma_f$  are at least equal to those corresponding to the rotational value  $\gamma_{f,2}$  (Eq. (11)) and thus to the final rotational kinetic energy of  $(720/2) \text{ MeV} = 360 \text{ MeV}$  (Eq. (15)).

We recall from (11) that

$$\gamma_{f,2} = 2^{1/3} \left( \frac{m_{Pl}}{m_0} \right)^{1/3} \quad (11)$$

for a two particle system, and from Eq. (16) that

$$\gamma_{f,3} = 3^{1/12} \left( \frac{m_{Pl}}{m_0} \right)^{1/3}, \quad (18)$$

for a three particle system (baryon). If the velocity is smaller, then  $\gamma$  will not be sufficiently large to satisfy the force balance (9) and thus to maintain a stable bound rotational state. Consequently, a necessary requirement for a successful association is that the initial particle velocity,  $v_0$ , and corresponding initial Lorentz factor,  $\gamma_0$ , is sufficiently large so that the Compton (or de Broglie) wavelengths of the two particles reach each other at  $r = r_f$  with at least the above  $\gamma_f$  value. The particle acceleration between the initial  $\gamma_0$  value to the final  $\gamma_f$  value must take place between the mean distance,  $r_0$ , between two particles. Since on earth there are approximately  $60 \times 10^9$  neutrinos crossing every  $\text{cm}^2$  per s, the latter distance ( $r_0$ ) can be estimated to be in the range  $r_0 = 10^{-3} \text{ m}$ . The minimum necessary  $\gamma_0$  value corresponds via the equation

$$E_0 = \gamma_0 m_0 c^2, \quad (19)$$

to the minimum energy which a neutrino must have at  $r = r_0$  and  $t = t_0$  for a successful approach. Consequently  $E_0$  expresses the activation energy of hadronization.

In order to compute  $\gamma_0$  and thus  $E_0$  for the desired  $\gamma_f$  values of equations (11) and (18), one starts from Fig. 5 using the relativistic one-dimensional equation of motion in conjunction with the gravitational force expression of Eq. (9), i.e.

$$\frac{d}{dt}(\gamma m_0 v) = \frac{G m_0^2 \gamma^6}{4 r^2}, \quad (20)$$

with initial conditions  $r = r_0$ ,  $v = v_0$  and  $\gamma = \gamma_0 = (1 - v_0^2/c^2)^{-1/2}$  at  $t = 0$ . The details of the derivation leading to the computation of  $\gamma_0$  in terms of  $\gamma_f$  and thus of the activation energy  $E_0 = \gamma_0 m_0 c^2$  are given in Appendix A, and here we only present the basic steps.

Introducing the dimensionless quantities

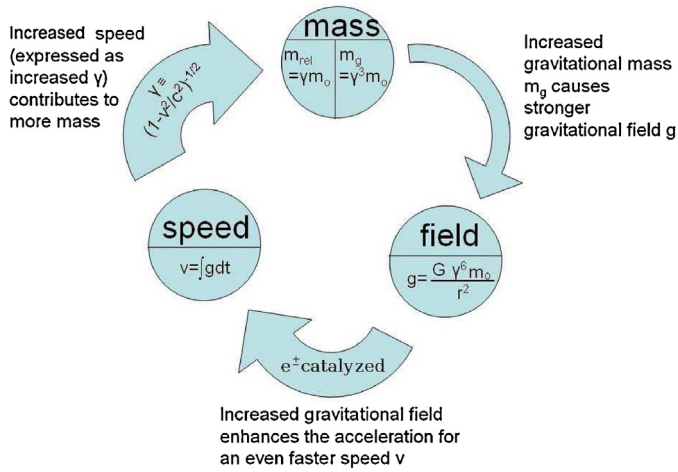
$$\tau = \frac{ct}{r_0}; \quad \beta = \frac{v}{c}; \quad x = \frac{r}{r_0}; \quad g_0 = \frac{G m_0}{4 c^2 r_0} = \frac{r_s}{8 r_0}, \quad (21)$$

and accounting for

$$\gamma = (1 - \beta^2)^{-1/2}; \quad \frac{dx}{d\tau} = -\beta, \quad (22)$$

one obtains after some algebra from Eq. (20) the expression,

$$\frac{d\gamma}{\gamma^6} = -g_0 \frac{dx}{x^2}, \quad (23)$$



**Fig. 6.** Autocatalysis of hadrogenesis and initially latent gravitational (Higgs) field (the autocatalytic cycle): acceleration increases the particle velocity, which increases its gravitational mass, which increases the field leading to further acceleration. The initially latent gravitational field has all the characteristics of the Higgs field, since it generates mass and it extends everywhere in space where neutrinos exist.

and finally that

$$\frac{\gamma}{\gamma_0} = \frac{1}{\left[1 + 5g_0\gamma_0^5 - \frac{5}{8}\gamma_0^5 \frac{r_s}{r}\right]^{1/5}} \approx \frac{1}{\left[1 - \frac{5}{8}\gamma_0^5 \frac{r_s}{r}\right]^{1/5}}, \quad (24)$$

where  $r_s = 2Gm_0/c^2 \ll r_f$ ,  $g_0 \ll 1$  and the approximation is valid for  $r \ll r_0$ . A plot of equation (24) for  $r_0 = 1$  mm and  $\gamma_0 = 5.754 \times 10^9$ , the minimum initial value required to achieve  $\gamma_f = 8.234 \times 10^9 (= 1.431\gamma_0)$ , is given by the solid curve in Fig. 5. One observes that when the two particles approach at a distance shorter than  $2 \times 5.5 \times 10^{-16}$  m, then there is a very pronounced increase in their Lorentz factor  $\gamma$  and their relativistic mass  $\gamma m_0$ .

Upon inserting  $r_s = 2Gm_0/c^2$  and  $r = \hbar/\gamma_{f2}m_0c$  or  $r = \hbar/\gamma_{f3}m_0c$  together with the definition of  $\gamma_{f2}$  (Eq. (11)) or  $\gamma_{f3}$  (Eq. (18)) in Eq. (24) (Appendix A), one obtains the simple result

$$\frac{\gamma_f}{\gamma_0} = 6^{1/5} = 1.431. \quad (25)$$

One may use Figs. 5–7 to visualize the autocatalysis involved in hadronization and the relation of this autocatalysis to the theoretically predicted mass generating Higgs field [24,25]:

At any point in time or space (Fig. 5) the gravitational field generated by the moving particle at a distance  $\ell$  from it is given by

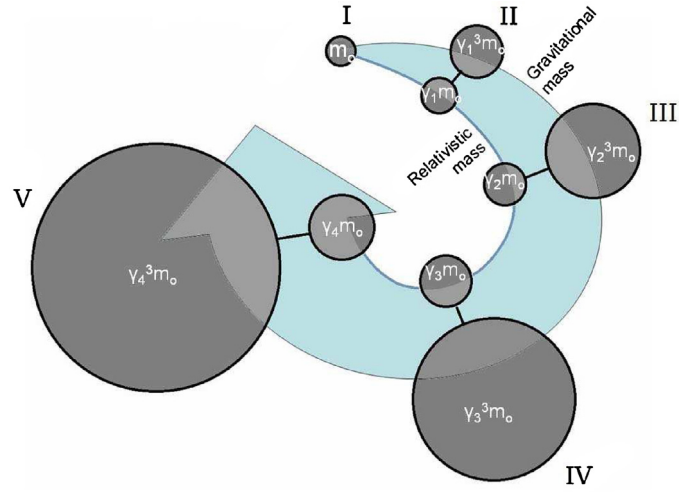
$$g = \frac{Gm_0\gamma^6}{\ell^2}. \quad (26)$$

On the other hand, the gravitational mass of the particle is

$$m_g = \gamma^3 m_0. \quad (27)$$

When the particle starts from rest, then  $\gamma = 1$  and the field is extremely weak, since gravitational interactions between slow neutrinos are totally negligible. As the particle gets slowly accelerated by this small field, its mass increases according to (27), thus  $\gamma$  increases, and reaches the initial value  $\gamma_0 = 5.75 \times 10^9$  shown in Fig. 5. At this point in time and space the field is still relatively weak. However, as the particle gets accelerated and  $\gamma$  increases, the relativistic mass increases as  $\gamma m_0$ , the gravitational mass increases as  $\gamma^3 m_0$  (Fig. 6) and the field increases as  $\gamma^6 m_0$ , Eq. (26) (Fig. 7). This further increases the particle speed and thus increases the mass, thus completing the autocatalytic cycle (Fig. 7).

This particle velocity-activated gravitational field has all the properties of the Higgs field [11,12,24,25]: it generates mass both



**Fig. 7.** Autocatalysis of hadrogenesis and initially latent gravitational (Higgs) field: Time evolution of neutrino relativistic and gravitational mass during its acceleration in the gravitational field generated by itself and by neighboring particles (I) Initially the gravitational and relativistic particle masses are practically equal to the rest mass  $m_0$ , thus are negligible, and the gravitational field is also negligible. (II) As the particle gets gradually accelerated, its relativistic and gravitational masses increase and so does the previously latent gravitational field. (III) As the particle gets further accelerated both its mass and the gravitation field strength increase dramatically and an increasing percentage of particles exceed the threshold activation energy for hadronization ( $\approx 251$  MeV in absence of  $e^\pm$ , 10 MeV in presence of  $e^\pm$ ). The initially latent gravitational field has all the characteristics of the Higgs field, since it generates mass and it extends everywhere in space where neutrinos exist.

for the accelerating particle and for all its neighboring particles which get attracted towards the initially accelerating particle. The field extends everywhere neutrinos exist, which is everywhere in space. Mass generation stops when all particles have been confined in massive hadrons, i.e. when mass has been generated. It thus appears that the Higgs field may be associated with or modeled as a latent gravitational field which gets activated by the presence of fast particles.

## 2.2. The catalytic role of positrons and electrons

Despite of the existence of the above potentially very efficient autocatalytic mechanism, Eq. (25) show that as long as  $r \ll r_0$ , one can expect only a 43.1% mass increase between two successive particle approaches. This shows that a catalyst is strongly needed to accelerate the process.

The catalytic role of positrons, or equivalently of electrons, in the hadronization process can be immediately recognized by replacing one of the two neutrinos in our model by a positron or by an electron, which has the same mass. In this case, the linear equation of motion (20) for the neutrino becomes

$$\frac{d(\gamma m_0 v)}{dt} = \frac{Gm_e m_0 \gamma^6}{4r^2}, \quad (28)$$

which, similarly to the derivation of (23) from (20), gives

$$\frac{d\gamma}{\gamma^6} = \frac{-\rho g_0 dx}{x^2}; \quad \rho = \frac{m_e}{m_0} = 1.17 \times 10^7, \quad (29)$$

where  $m_e$  is the rest mass of the electron ( $0.511 \times 10^6$  eV/ $c^2$ , Table 1).

Similarly to Eqs. (23) and (25) the solution of Eq. (29) is given by

$$\frac{\gamma_{f,e}}{\gamma_0} = (1 + 5\rho) = 35.76, \quad (30)$$

**Table 5**

Catalytic effect of positrons and electrons in hadronization (Temperatures are computed via  $T = \text{Energy}/k_b$  with  $k_b = 8.617 \times 10^{-5}$  eV/K, the Boltzmann constant).

	Value of $\gamma_f$	Necessary (Eq. (25)) value of $\gamma_0$ in absence of $e^\pm$	Necessary (Eq. (30)) value of $\gamma_0$ for $\rho_m = 25$ in presence of $e^\pm$
Meson	$8.234 \times 10^9$ $T = 4.18 \times 10^{12}$ K	$5.75 \times 10^9$ ( $\gamma m_0 c^2 = 251$ MeV) $T_0 = 2.92 \times 10^{12}$ K	$2.3 \times 10^8$ ( $\gamma m_0 c^2 = 10$ MeV) $T_0 = 1.16 \times 10^{11}$ K
Baryon	$7.163 \times 10^9$ [13,15] $T = 3.63 \times 10^{12}$ K	$5.01 \times 10^9$ ( $\gamma m_0 c^2 = 219$ MeV) $T_0 = 2.54 \times 10^{12}$ K	$2.0 \times 10^8$ ( $\gamma m_0 c^2 = 8.8$ MeV) $T_0 = 1.02 \times 10^{11}$ K

where we use the subscript “e” to denote the catalytic action of a positron or electron.

Thus, for any given  $\gamma_0$  value, one may define a mass generation enhancement ratio  $\rho_m$  via

$$\rho_m = \frac{\gamma_{f,e}}{\gamma_f} = \left(\frac{5\rho}{6}\right)^{1/5} = 25.0, \quad (31)$$

which implies that the presence of an electron or positron enhances by a factor of 25 the mass generated between two successive particle approaches. Also, one may use Eq. (30) to compute the reduction in activation energy for achieving a given mass value.

Thus, for  $\gamma_{f,2} = 8.234 \times 10^9$ , it follows from Eq. (30) that

$$\gamma_0 = 2.30 \times 10^8, \quad (32)$$

and thus

$$\gamma_0 m_0 c^2 = 1.00 \times 10^7 \text{ eV} = 10 \text{ MeV}, \quad (33)$$

which is a factor of 25 smaller than the activation energy (251 MeV) in the absence of an electron or positron. Consequently, the catalytic role of the positron (or electron) leads to a significant decrease in the necessary initial energy  $\gamma m_0 c^2$  for reaching  $\gamma_f$  and thus for achieving hadronization.

Table 5 shows the corresponding values of  $\gamma_0$  for achieving hadronization for the formation of two-particle hadrons (mesons) and of three-particle hadrons (baryons). It is worth noting that the temperature for the quark-gluon plasma condensation to form hadrons (transition temperature) computed from quantum chromodynamics (QCD) is 173 MeV ( $\approx 2 \times 10^{12}$  K) [14] in good qualitative agreement with Table 5.

One observes that there is a twentyfold (19.88) decrease in the computed activation temperature and activation energy (Tables 5 and 6).

In summary, due to their significantly larger mass than that of neutrinos, positrons and electrons can act as very efficient catalysts for hadronization. They strongly attract neutrinos, first by guiding them via charge-induced dipole interactions, and then via the much stronger gravitational force creating  $W^\pm$  bosons (Fig. 4), as well as heavier  $Z^0$  and Higgs bosons which are rotating triangular  $e^+ - \nu_e - e^-$  and tetrahedral  $e^+ - \bar{\nu}_e - \nu_e - e^-$  structures respectively [17,18,26]. This is the first step for baryogenesis and, more generally, for hadronization (Fig. 4). All these bosons are short-lived with lifetimes below  $10^{-22}$  s [11,12,17,18,26], and produce via their decay highly energetic neutrinos which are very

**Table 6**

Catalysis of hadronization: effect of positrons and electrons on the activation energy of hadronization and comparison with  $\text{NH}_3$  synthesis [5].

	Value of activation energy		
	Meson synthesis	Baryon synthesis	Ammonia synthesis
Without $e^\pm$	0.251 GeV	0.265 GeV	1.226 eV
With $e^\pm$	0.010 GeV	0.0105 GeV	0.509 eV

effective for hadronization [17,18,26]. This suggests the following catalytic action of the above bosons and of the positrons and electrons which generate then: they accelerate via their gravitational attraction neutrinos to very high speeds and thus they lower significantly the activation energy for hadronization [17,18,26], Table 5. It is worth noting that the catalytic role of positrons in the synthesis of baryons is also manifested by their presence at the center of the rotating neutrino ring [13], depicted schematically in Fig. 3.

Table 6 compares the effect of  $e^\pm$  on the activation energies of hadronization and ammonia synthesis. One observes that the effect on hadronization is significantly stronger.

### 2.3. Kinetics of baryogenesis

The rate of hadronization is determined by the number of successful collisions per unit volume and time. In gas phase chemical kinetics the collision frequency  $z$  is determined by the expression

$$z = N_A \sigma_{AA} \left(\frac{8k_b T}{\pi m}\right)^{1/2} \exp\left(\frac{-E_a}{RT}\right), \quad (34)$$

where  $N_A$  (particles/m<sup>3</sup>) is the concentration of neutrinos and the term  $(8k_b T/\pi m)^{1/2}$  expresses an average particle velocity. In the present case, the particles have velocities very close to  $c$ , thus Eq. (34) becomes

$$z = N_A \sigma_{AA} c \exp\left(\frac{-E_a}{RT}\right). \quad (35)$$

The addition of positrons or electrons can affect  $z$  in two different ways: first by modifying the reaction cross section  $\sigma_{AA}$  to  $\sigma_{AB}$ , and second by affecting the activation energy  $E_a$ . As already shown in the previous section, the addition of electrons or positrons causes a dramatic 25-fold decrease in the activation energy of baryosynthesis, namely from 251 MeV to 10 MeV. This implies that a baryosynthesis reaction can take place at temperatures as low as  $10^{11}$  K. At this temperature the presence of electrons or positrons enhances the rate of baryosynthesis by a factor of the order  $10^{10}$ , actually by a factor of  $\exp[(251/10) - 1] = 2.6 \times 10^{10}$ .

### 3. Analogies between the catalysis of proton synthesis and of chemical synthesis

The previous sections have shown that there exist several similarities between hadronization and chemical synthesis. One may distinguish three types of analogies.

- Obvious parallels
- parallels which are not obvious and deserve some discussion
- points where the analogy breaks down

Obvious parallels are the following:

- 1 The strong similarity between the homogeneous oxidation of  $\text{H}_2$  to  $\text{H}_2\text{O}$ , catalyzed by collisions involving OH radicals and  $\text{O}_2$  [2,3], and the conversion of neutrinos to protons and other hadrons, catalyzed by collisions or near-collisions involving electrons, or positrons, and neutrinos. In both cases there is significant autocatalysis. In  $\text{H}_2$  oxidation the OH radicals are generated in the reacting mixture. In hadronization, high speed generates mass which strengthens the field which enhances speed.
- 2 The strong exothermicity of both processes which leads to the same expression for the adiabatic temperature rise, i.e.  $\Delta T = (-\Delta H)/\bar{C}_p$ , or  $\Delta T = (-\Delta U)/\bar{C}_v$  where  $\bar{C}_p$  and  $\bar{C}_v$  are in both cases of the order 30 J/mol·K. This is because both neutrinos and baryons can be reasonably assumed to behave approximately as ideal gases. Thus each of the three neutrinos has three

translational degrees of freedom and each baryon has three translational, three rotational and possibly three vibrational degrees of freedom, each contributing  $R/2$  to the  $C_v$  of the gas mixture components, as in all ideal gases.

- 3 The fact that this exothermicity can lead in both cases to explosions; it is conceivable that the “big bang” was simply such an explosion caused by the autothermal and autocatalytic hadronization reaction occurring in our universe, if it was filled with neutrinos, as it is more or less today but with significantly higher densities and thus shorter interparticle distances.
- 4 The pronounced lowering in the activation energy via the presence of electrons and protons or positrons (Table 6), and the concomitant dramatic increase in the rate of the reaction.

As already noted, the analogy between homogeneous  $H_2$  oxidation, catalyzed by OH radicals, and the hadronization reaction catalyzed by  $e^\pm$  is quite strong. The only significant difference lies in the mechanism of autocatalysis. In  $H_2$  oxidation the catalyst, OH, is produced from the reactants. In hadronization, the catalytic rate is enhanced by the velocity-enhanced gravitational mass of the reactants.

The analogies between ammonia synthesis and hadronization are more phenomenological and require some additional discussion:

Electrons (and protons or positrons) enhance the rate both of ammonia synthesis and of hadronization and they both cause a significant decrease in activation energy (Table 6). Despite this phenomenological strong similarity, significant differences exist in the promoting/catalytic mechanism:

- i. In ammonia synthesis, electrons (or protons) are promoters of the metal-based synthesis catalysts. In hadronization, electrons (or positrons) can be the catalyst itself, or a major part of it, as an important component of bosons (Table 3).
- ii. In ammonia synthesis the action of the electron (or protons) is electrostatic. They modify, via Coulombic interactions, the strength of the chemisorptive bonds of reactants, intermediates and products.
- iii. In hadronization, electrons (or positrons) are not promoters, but are the catalyst or part of the catalyst itself, with a role similar to that of OH radicals in  $H_2$  oxidation (Table 3).

#### 4. Conclusion

Hadronization exhibits several thermodynamic and kinetic similarities both with homogeneous gas phase oxidation reactions, which are catalyzed by OH radicals, and with chemical synthesis reactions such as ammonia synthesis, which is catalyzed by transition metal catalysts doped by electropositive or electronegative promoters.

It appears that the catalytic role of electrons and positrons and also of the bosons they form [17,18] in hadronization, is at least as essential and pronounced as the role of OH radicals in homogeneous oxidation catalysis and as the role of anionic or cationic promoters in heterogeneous chemical synthesis catalysis: it leads to a 25-fold decrease in activation energy (Table 6) and to a concomitant  $2.65 \times 10^{10}$  rate enhancement at  $T = 1.1 \times 10^{11}$  K, i.e. to a 25-fold decrease in operating temperature (Table 5).

This very pronounced catalytic effect is mainly due to the much higher rest mass of an electron or positron ( $0.511 \text{ MeV}/c^2$ ) vs a neutrino ( $\sim 0.0437 \text{ eV}/c^2$ ), although electric charge-induced dipole interactions can also have a secondary role as in the case of bosons formation [17,18]. On the other hand, another interesting role of electrons and positrons in baryogenesis is the fractional charges they can induce on the electrically polarizable neutrinos.

It is worth recalling that in heterogeneous catalysis the catalytic role of electrons and protons is due to the chemical (electrostatic) interactions of the electrons with the reactants and products. In hadronization, these electrostatic forces have a secondary role by inducing partial charges on neutrinos and thus by attracting and guiding them to successful collisions, but the dominant role is due to their much heavier mass, a factor of  $10^7$  heavier than that of neutrinos, which leads to a 25-fold decrease in hadronization activation energy and to a very large ( $10^{10}$ ) enhancement in the rate of hadronization.

#### Acknowledgements

We thank Professor Gary Haller for valuable suggestions and Professors Xenophon Verykios, Chris Tully and Stefanos Aretakis for helpful discussions over the years.

#### Appendix A. Computation of the activation energy for hadronization

Here we formulate and solve Eq. (20) which refers to Fig. 5 and leads to Eqs. (24) and (25). The definition of the Lorentz factor  $\gamma$ , namely

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \quad (\text{A.1})$$

implies

$$\frac{d\gamma}{dt} = \frac{\gamma^3}{c^2} v \frac{dv}{dt}. \quad (\text{A.2})$$

Hence,

$$\frac{d}{dt}(m_0 \gamma \dot{r}) = m_0 \gamma \ddot{r} + \frac{m_0 \gamma^3}{c^2} v \frac{dv}{dt} \dot{r}. \quad (\text{A.3})$$

Incidentally, it is shown in [27] that

$$v \frac{dv}{dt} = \dot{r} \cdot \ddot{r},$$

and then the RHS of Eq. (A.3) coincides with the following expression found in most books [20]:

$$\frac{d}{dt}(m_0 \gamma \dot{r}) = m_0 \gamma \ddot{r} + \frac{m_0 \gamma^3}{c^2} (\dot{r} \cdot \ddot{r}) \dot{r}. \quad (\text{A.4})$$

We consider two particles each of rest mass  $m_0$ , located initially at  $r_0$  and  $-r_0$  respectively, and with initial speeds  $v_0$  (Fig. 5). Using

$$\underline{r} = r \underline{i}, \quad \dot{\underline{r}} = -v \underline{i}, \quad \ddot{\underline{r}} = -\frac{dv}{dt} \underline{i}, \quad (\text{A.5})$$

Eq. (A.3) becomes

$$\frac{d}{dt}(m_0 \gamma v) = m_0 \gamma \frac{dv}{dt} + \frac{m_0 \gamma^3}{c^2} v^2 \frac{dv}{dt}.$$

Hence, the basic equation of motion, i.e. Eq. (20)

$$\frac{d}{dt}(m_0 \gamma v) = \frac{G m_0^2 \gamma^6}{4 r^2}, \quad (\text{A.6})$$

yields

$$m_0 \gamma \left(1 + \gamma^2 \frac{v^2}{c^2}\right) \frac{dv}{dt} = \frac{G m_0^2 \gamma^6}{4 r^2}. \quad (\text{A.7})$$

Eq. (A.1) implies

$$1 + \gamma^2 \frac{v^2}{c^2} = \gamma^2,$$

thus Eq. (A.7) becomes

$$\frac{dv}{dt} = \frac{G m_0 \gamma^3}{4 r^2}. \quad (\text{A.8})$$



We introduce the following dimensionless quantities:

$$\tau = \frac{ct}{r_0}, \quad \beta = \frac{v}{c}, \quad x = \frac{r}{r_0}, \quad g_0 = \frac{Gm_0}{4c^2 r_0}. \quad (\text{A.9})$$

Then, Eqs. (A.1) and (A.8) become

$$\gamma = (1 - \beta^2)^{-1/2}, \quad (\text{A.10})$$

and

$$\frac{d\beta}{d\tau} = \frac{g_0 \gamma^3}{x^2}. \quad (\text{A.11})$$

Furthermore, the dimensionless form of the equation

$$\begin{aligned} \frac{dr}{dt} &= -v, \\ \text{is} \\ \frac{dx}{d\tau} &= -\beta. \end{aligned} \quad (\text{A.12})$$

Hence, using Eqs. (A.11) and (A.12) we find

$$\frac{d\beta}{dx} = \frac{d\beta}{d\tau} \frac{d\tau}{dx} = - \left( \frac{g_0 \gamma^3}{x^2} \right) \frac{1}{\beta}. \quad (\text{A.13})$$

Eq. (A.10) implies  $\beta d\beta = \gamma^{-3} d\gamma$ , and then Eq. (A.13) becomes

$$\frac{d\gamma}{\gamma^6} = -g_0 \frac{dx}{x^2}. \quad (\text{A.14})$$

Supplementing this equation with the condition  $\gamma = \gamma_0$  at  $x = 1$ , we find

$$\gamma^{-5} - \gamma_0^{-5} = 5g_0 \left( 1 - \frac{1}{x} \right). \quad (\text{A.15})$$

Noting that

$$\frac{g_0}{x} = \frac{Gm_0}{4c^2 r} = \frac{1}{8} \frac{r_s}{r}, \quad r_s = \frac{2Gm_0}{c^2}, \quad (\text{A.16})$$

where  $r_s$  denotes the Schwarzschild radius of the body of mass  $m_0$ , Eq. (A.15) becomes

$$\frac{\gamma}{\gamma_0} = \frac{1}{\left( 1 + 5g_0 \gamma_0^5 - \frac{5}{8} \gamma_0^5 \frac{r_s}{r} \right)^{1/5}}. \quad (\text{A.17})$$

For  $g_0 \ll 1$ , i.e. for  $r_s \ll r_0$  which is indeed the case, and for  $r \ll r_0$ , Eq. (A.17) becomes

$$\frac{\gamma}{\gamma_0} = \frac{1}{\left[ 1 - \frac{5}{8} \gamma_0^5 \frac{r_s}{r} \right]^{1/5}}, \quad (\text{A.18})$$

which is Eq. (24).

Upon substituting in (A.15)  $r_s$  from (A.16) and  $r$  from the de Broglie wavelength expression (13), we find

$$\begin{aligned} \left( \frac{\gamma_{f,2}}{\gamma_0} \right)^5 &= \frac{1}{1 - \frac{5}{4} \gamma_0^5 \frac{Gm_0^2}{\hbar^2} \gamma_{f,2}} = \frac{1}{\left[ 1 - \frac{5}{4} \left( \frac{m_0^2}{m_{Pl}^2} \right) \gamma_{f,2} \gamma_0^5 \right]^{1/5}} \\ &= 4 \frac{m_{Pl}^2}{m_0^2}, \end{aligned} \quad (\text{A.19})$$

Substituting from Eq. (11) Eq. (A.19) becomes

$$\left( \frac{\gamma_{f,2}}{\gamma_0} \right)^5 = \frac{1}{1 - 5 \left( \frac{\gamma_0}{\gamma_{f,2}} \right)}. \quad (\text{A.20})$$

Therefore

$$\left( \frac{\gamma_{f,2}}{\gamma_0} \right)^5 = 6; \quad \frac{\gamma_{f,2}}{\gamma_0} = 6^{1/5} = 1.431, \quad (\text{A.21})$$

which is Eq. (25). From the desired  $\gamma_{f,2}$  value of  $8.234 \times 10^9$  one computes  $\gamma_0 = 5.754 \times 10^9$ , thus  $E_0 = \gamma_0 m_0 c^2 = 251 \text{ MeV}$ .

## References

- [1] D. Green, R. Perry, *Perry's Chemical Engineers' Handbook*, 8th ed., McGraw-Hill Companies, 2008, ISBN-13: 978-0071422949.
- [2] G.L. Schott, J.L. Kinsey, Kinetic studies of hydroxyl radicals in shock waves. II. Induction times in the hydrogen-oxygen reaction, *J. Chem. Phys.* 29 (1958) 1177–1182.
- [3] B. Lewis, *Combustion, Flames and Explosions of Gases*, Academic Press, Orlando, 1987.
- [4] H. Liu, *Ammonia Synthesis catalysts: Innovation and Practice*, World Scientific, Singapore, 2013.
- [5] M. Kitano, S. Kanbara, Y. Inoue, N. Kuganathan, P.V. Sushko, T. Yokoyama, M. Hara, H. Hosono, Electride Support boosts nitrogen dissociation over ruthenium catalyst and shifts the bottleneck in ammonia synthesis, *Nat. Commun.* 6 (2015) 6731.
- [6] C.G. Yiokari, G.E. Pitselis, D.G. Polydoros, A.D. Katsaounis, C.G. Vayenas, High-pressure electrochemical promotion of ammonia synthesis over an industrial iron catalyst, *J. Phys. Chem.* 104 (2000) 10600–10602.
- [7] Ph. Vernoux, L. Lizzaraga, M.N. Tsampas, F.M. Sapountzi, A. De Lucas-Consuegra, J.-L. Valverde, S. Souentie, C.G. Vayenas, D. Tsiplakides, S. Balomenou, E.A. Baranova, Ionically conducting ceramics as active catalyst supports, *Chem. Rev.* 113 (2013) 8192–8260.
- [8] C.G. Vayenas, S. Bebelis, C. Pliangos, S. Brosda, D. Tsiplakides, *Electrochemical Activation of Catalysis: Promotion, Electrochemical Promotion and Metal-Support Interactions*, Kluwer/Plenum Press, New York, 2001.
- [9] J. Nicole, D. Tsiplakides, C. Pliangos, X.E. Verykios, Comminellis Ch, C.G. Vayenas, Electrochemical promotion and metal-support interactions, *J. Catal.* 204 (2001) 23–34.
- [10] G.L. Haller, New catalytic concepts from new materials: understanding catalysis from a fundamental perspective, past, present, and future, *J. Catal.* 216 (2003) 12–22.
- [11] D. Griffiths, *Introduction to Elementary Particles*, 2nd ed., Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim, 2008.
- [12] C.G. Tully, *Elementary Particle Physics in a Nutshell*, Princeton University Press, 2011, ISBN13: 978-0-691-13116-0.
- [13] C.G. Vayenas, S. Souentie, Gravity, Special Relativity and the Strong Force: A Bohr-Einstein-de-Broglie Model for the Formation of Hadrons, Springer, New York, 2012, ISBN 978-1-4614-3935-6.
- [14] P. Braun-Munzinger, J. Stachel, The quest for the quark-gluon plasma, *Nature* 448 (2007) 302–309.
- [15] C.G. Vayenas, S. Souentie, A. Fokas, A Bohr-type model with gravity as the attractive force, 2013, arXiv:1306.5979v1 [physics.gen-ph]; (2014) *Physica A* 405:60–379.
- [16] C.G. Vayenas, Mathematical modeling of mass generation via confinement of relativistic particles, *J. Phys.* 490 (2014) 012084, <http://dx.doi.org/10.1088/1742-6596/490/1/012084>.
- [17] C.G. Vayenas, A.S. Fokas, D. Grigoriou, On the structure, masses and thermodynamics of the  $W^\pm$  bosons, *Physica A* 450 (2016) 37–48.
- [18] A.S. Fokas, C.G. Vayenas, On the structure, mass and thermodynamics of the  $Z^0$  bosons, *Physica A* 464 (2016) 231–240.
- [19] A.P. French, *Special Relativity*, W.W. Norton and Co, New York, 1968.
- [20] J. Freund, *Special Relativity for Beginners*, World Scientific Publishing, Singapore, 2008.
- [21] A. Einstein, Zür Elektrodynamik bewegter Körper, *Ann. der Physik*. Bd. XVII. S. 17 (1905) (1923) 891–921, English translation On the Electrodynamics of Moving Bodies (<http://fourmilab.ch/etexts/einstein/specrel/www/>) by G.B. Jeffery and W. Perrett.
- [22] R.N. Mohapatra, et al., Theory of neutrinos: a white paper, *Rep. Prog. Phys.* 70 (2007) 1757–1867.
- [23] H. Fritzsch, Neutrino masses and flavor mixing, *Modern Phys. Lett. A* 30 (16) (2015), 1530012(1–6).
- [24] P. Higgs, Broken symmetries and the masses of Gauge Bosons, *Phys. Rev. Lett.* 13 (16) (1964) 508–509, <http://dx.doi.org/10.1103/PhysRevLett.13.508>, Bibcode:1964PhRvL.13.508H.
- [25] Higgs Boson, <https://en.wikipedia.org/wiki/Higgsboson>.
- [26] C.G. Vayenas, A.S. Fokas, D. Grigoriou, On the structure, mass and thermodynamics of Higgs bosons, 2016, submitted for publication.
- [27] A.S. Fokas, C.G. Vayenas, D. Grigoriou, Analytical computation of the Mercury perihelion precession via the relativistic gravitational law and comparison with general relativity, 2015, arXiv:1509.03326v1 [gr-qc].